Measurement and noise characterization of optically induced index changes using THz interferometry

J. A. Small and R. A. Cheville
School of Electrical and Computer Engineering, Oklahoma State University, Stillwater, Oklahoma 74078

(Received 22 December 2003; accepted 6 April 2004; published online 7 May 2004)

A Michelson interferometer designed for broadband single-cycle THz pulses is used to characterize optically induced index changes in semiconductors which result in submicron changes in optical path length. The interferometric measurements are compared both to standard THz time-domain spectroscopy (THz-TDS) and differential THz-TDS based on modulation of the sample. By analyzing noise contributions in THz spectroscopy systems, it is shown that the destructive interference achieved in THz interferometry reduces both some sources of random errors as well as errors due to system drift. © 2004 American Institute of Physics. [DOI: 10.1063/1.1758292]

Advances in optical generation and detection of near-single-cycle THz pulses have resulted in ways to measure material properties in the hard-to-access far-infrared spectral region. A variety of characterization techniques based on THz time-domain spectroscopy (THz-TDS) have been developed to measure materials as diverse as semiconductors,4 flames,5 and water content.6 In THz-TDS, a single-cycle electromagnetic pulse with THz bandwidth is used to determine the refractive index and absorption of a sample by comparing the pulse transmitted through the sample to a reference pulse transmitted through dry air. Since THz pulses are near-single cycle with subpicosecond duration, measurement bandwidths can extend from less than 100 GHz to 5 THz.

Because THz-TDS measures transmission changes in a broadband spectrum, it is difficult to accurately characterize materials when the sample is thin, or with low index and/or absorption. When \( k(\omega) d < 1 \) —with \( k(\omega) \) as the complex frequency dependent wave constant and \( d \) as the sample thickness, noise or system drift lead to errors which are greater than the small changes being measured. Recently, several techniques have been developed to characterize thin samples at THz frequencies. One of these is differential time domain spectroscopy (DTDS), which dithers a film on a substrate in and out of the THz beam. By moving the sample so that the boundary of the film moves across the beam, only the modulation induced on the beam by the film is detected. DTDS has been used to determine the refractive index of a 300 nm thick parylene-N film4 and other thin or low index films.4-6 By using a double-modulation technique in which both the sample and the THz beam itself are modulated and measured using two lock-in amplifiers,7 films as thin as 100 nm have been characterized. DTDS substantially increases sensitivity over standard TDS with the primary difficulty ensuring homogeneity of the sample and substrate.

Interferometric techniques have also been used to measure small changes to a THz beam.8,9 In this case, destructive interference occurs between simultaneously measured sample and reference pulses. By eliminating the background signal, some noise sources and drift are eliminated. In this letter, we demonstrate THz interferometric characterization of optically induced index changes in a high resistivity silicon wafer, permitting direct comparisons between THz interferometry, THz-TDS, and DTDS. Both THz interferometry and DTDS give comparable results. Following previous treatments of noise sources in THz-TDS,10 improvements in resolution are shown to cause a reduction in emitter noise compared to THz-TDS.

The experimental setup consists of a THz-TDS system1 configured as a Michelson interferometer,8 shown in Fig. 1. The THz emitter is a coplanar transmission line fabricated on semi-insulating GaAs and biased at 80 V dc, which generates freely propagating pulses of THz electromagnetic radiation.1 The incident optical excitation beam is modulated at \( f_{\text{THz}} = 1650 \) Hz by a mechanical chopper. The generated THz radiation is collimated by a silicon lens and parabolic mirror. A 1.0 mm thick n-type high resistivity (>10 kΩ cm) silicon wafer is used to generate the THz pulses while optical excitation of the sample is using a HeNe laser at 632 nm.
The output from the first lock-in is sent to a integration time of 1 ms, and is used to measure the incident numerical Fourier transforms of the THz pulses transmitted first lock-in amplifier has a reference frequency of \( f \). The time-resolved electric field of the superposition in the dipole is measured using a current amplifier and induced change in transmission of the THz pulse.

To demonstrate that THz interferometry can distinguish through the sample arm, A, with and without optical illumination. For THz-TDS the complex transmission coefficient is defined as \( T(\omega) = S(\omega) / R(\omega) = \rho(\omega) e^{i \psi(\omega)} \), from which the real part of the complex refractive index, \( n(\omega) = n'(\omega) - i n''(\omega) \), is extracted from the argument, \( \psi(\omega) \), and the absorption coefficient, \( \alpha(\omega) = \omega n''(\omega) / c \), from the modulus, \( \rho(\omega) \).

Errors in the measurement of \( n(\omega) \) are proportional to the variance of the modulus, \( \sigma_{s}^{2}(\omega) \), which in turn is related to the variance of the reference, \( \sigma_{R}^{2}(\omega) \), and sample, \( \sigma_{s}^{2}(\omega) \), data scans:

\[
\sigma_{s}^{2}(\omega) = \frac{\sigma_{R}^{2}(\omega)}{|R(\omega)|^2} + \frac{\rho(\omega)}{|R(\omega)|^2} \sigma_{R}^{2}(\omega). \tag{1}
\]

As in the case here, when \( \rho(\omega) \) approaches unity:

\[
\sigma_{s}^{2}(\omega) = 2 \frac{\sigma_{\text{noise}}^{2}(\omega)}{|R(\omega)|^2}. \tag{2}
\]

Noise contributions on the sample or reference scans arise from three sources:

\[
\sigma_{\text{noise}}^{2}(\omega) = \rho(\omega) \sigma_{R}^{2}(\omega) + \sigma_{s}^{2}(\omega) + \sigma_{R}^{2}(\omega) \tag{3}
\]

where for thin sample \( \rho(\omega) \approx 1 \) and small absorption. These three terms correspond to noise from the THz emitter \( \sigma_{R}^{2}(\omega) \), shot noise in the THz detector \( \sigma_{s}^{2}(\omega) \), and other signal independent noise sources, \( \sigma_{\text{noise}}^{2}(\omega) \) such as laser noise, electronic noise, and Johnson noise. The shot noise can be written as \( \sigma_{\text{noise}}^{2}(\omega) = 2 e \Delta f X(\omega) \) with \( e \) the electron charge, \( \Delta f \) the measurement bandwidth determined by the lock-in integration time, and \( X(\omega) \) either \( R(\omega) \) or \( S(\omega) \). As shown in Ref. 10, the dominant noise term comes from the THz emitter, \( \sigma_{R}^{2}(\omega) \), especially in the case of slightly absorbing samples with \( \rho(\omega) \) approaching unity, and \( S(\omega) \approx R(\omega) \). For THz-TDS, the sample and reference scans are measured independently and the noise on the sample and reference scans are uncorrelated.

For THz interferometry, a more relevant noise figure is given by the variance of the measured interference signal, \( D(\omega) = R(\omega) - S(\omega) \). Since in THz interferometry the difference is obtained in a single measurement, the emitter noise is correlated between the sample arm (A) and reference arm (B). The noise on the interferometric signal can thus be written:

\[
\sigma_{s}^{2}(\omega) = (1 - \rho(\omega))^2 \sigma_{R}^{2}(\omega) + 2 e \Delta f R(\omega)(1 - \rho(\omega)) + 2 \sigma_{R}^{2}(\omega). \tag{4}
\]

For \( k(\omega) d \ll 1 \), the modulus of the difference signal, \( 1 - \rho(\omega) \), is proportional to \( |\delta n(\omega)|/R(\omega) \) in the case of transparent, \( n'(\omega) \approx n''(\omega) \), or lossy, \( n'(\omega) \ll n''(\omega) \), samples. A comparison of Eq. (3) with Eq. (4) shows that the effects of both the shot noise and emitter noise are substantially reduced in THz interferometry. The previous analysis assumes ideal cancellation of the two arms of the interferometer; in an actual experimental configuration interference of the sample and reference pulses can achieve approximately 95% amplitude cancellation due to lack of rotational symmetry of the THz beam.8,13

To demonstrate that THz interferometry can distinguish small changes in a sample, we utilize optical modulation of the refractive index of a high resistivity, \( > 10 \text{ k}\Omega \text{ cm} \) silicon wafer, which is shown in Fig. 2; the inset shows the measurement bandwidth determined by the lock-in integration time, and \( X(\omega) \) either \( R(\omega) \) or \( S(\omega) \). As shown in Ref. 10, the dominant noise term comes from the THz emitter, \( \sigma_{s}^{2}(\omega) \), especially in the case of slightly absorbing samples with \( \rho(\omega) \) approaching unity, and \( S(\omega) \approx R(\omega) \). For THz-TDS, the sample and reference scans are measured independently and the noise on the sample and reference scans are uncorrelated.

For THz interferometry, a more relevant noise figure is given by the variance of the measured interference signal, \( D(\omega) = R(\omega) - S(\omega) \). Since in THz interferometry the difference is obtained in a single measurement, the emitter noise is correlated between the sample arm (A) and reference arm (B). The noise on the interferometric signal can thus be written:

\[
\sigma_{s}^{2}(\omega) = (1 - \rho(\omega))^2 \sigma_{R}^{2}(\omega) + 2 e \Delta f R(\omega)(1 - \rho(\omega)) + 2 \sigma_{R}^{2}(\omega). \tag{4}
\]
masses

sity N assumed equal for electrons and holes, and effective
results in a carrier density of

DTDS scan has been normalized to the throughput of the
since the carrier recombination time of

sume that the carrier density across the sample is uniform

electron or hole plasma frequency given by

where

\( \varepsilon_e = \varepsilon_i = 11.7 \) is the relative permittivity of Si, \( \omega_p \) is the
electron or hole plasma frequency given by \( \omega_p^2 = N e^2 / \varepsilon_0 m_{e,h}^* \) with free-space
permittivity \( \varepsilon_0 \), carrier density \( N \) assumed equal for electrons and holes, and effective
masses \( m_{e,h}^* = 0.26 \) and \( m_{e,h}^* = 0.37 \). \( \Gamma \) is the carrier damping
rate \( 1 / \tau_{e,h} \). For the >10 kΩ cm Si sample, the only appreciable
contributions to the free carrier density are those
 carriers generated by optical modulation. For these measure-
ments, the excitation intensity used, \( 3.58 \times 10^{-2} \) mW/cm²,
results in a carrier density of \( N = 6.53 \times 10^{12} \) cm⁻³. We
assume that the carrier density across the sample is uniform
since the carrier recombination time of \( \tau_{e,h} = 25 \) ms corre-
sponds to a diffusion length greater than 2 mm at low carrier
densities.¹⁶

Figure 3(a) compares the difference between two con-
secutive THz scans taken with and without optical excitation
(dotted line), a single data scan taken using DTDS (dashed
line), and a scan on the THz interferometer (solid line) with
the constant background signal numerically subtracted.⁸
The DTDS scan has been normalized to the throughput of the
THz interferometer (34.7%). The ratio between peak signal
and background noise of the difference signal is 4.9:1. For
the data scan using the THz interferometer, the ratio is 21:1
while that of the DTDS measurement is 22:1.

Figure 3(b) plots the measured optically induced change
in refractive index, \( \Delta n(\omega) / n(\omega) \), obtained from THz- 
ferogram (points) and that determined by THz-TDS (open
circles), both using the Fourier transform of the temporal
data of Fig. 3(a) where

\[
\Delta(n_\omega(\omega) - n_{\omega}(\omega)) = -i \frac{c}{\omega d} \ln \left( \frac{R(\omega) - D(\omega)}{R(\omega)} \right). 
\]  

The index change calculated using Drude theory is shown as
a solid line in Fig. 3(b). The measured index change was
determined taking into account the Fresnel transmission
coefficients on the input and output faces of the Si wafer, but
not the Fabry–Perot term since reflections from the wafer
faces were time gated out. Using THz interferometry, index
changes of \( 10^{-4} \) can be measured, which corresponds to a
change in the optical path length of 342 nm. The same mea-
surement made using THz-TDS only closely approximates
the actual index change over a small frequency range. At 0.7
THz, the peak of the THz amplitude spectrum, 342 nm is an
optical path length change of \( \lambda/1250 \). Extrapolating the re-

cults to a unity signal-to-noise ratio, the measurement limit
of the system is 32 nm or \( \lambda/13 \).⁵⁰

We have demonstrated that THz time-domain inter-
ferometry can measure photoinduced refractive index change
at THz frequencies of \( \Delta n n < 10^{-4} \), and demonstrated sensiti-

ty to changes in optical path length comparable to those
previously obtained with DTDS.⁶ For thin-film measure-
ments, DTDS utilizes spatial dithering of the THz beam
across a step boundary of the film while THz interferometry
needs an identical reference substrate. Analysis of noise
sources of THz-TDS, fully treated in Ref. 10, show that the
measurement sensitivity arises from a reduction of both
emitter and shot noise.

The authors would like to acknowledge support from the
National Science Foundation (ECS-9984896), Army
Research Office (DAAD19-99-R-BAA8), and Department of
Energy. One of the authors (J.S.) acknowledges support of
the NSF IGERT program for support during this work.

7. M. Brucheler, P. H. Bolivar, and H. Kurz, Appl. Phys. Lett. 81, 1791
(2002).
78, 835 (2001).
(2000).
(2003).