# Effect of metal permittivity on resonant properties of terahertz metamaterials

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We investigate the effect of metal permittivity on resonant transmission of metamaterials by terahertz time-domain spectroscopy. Our experimental results on double split-ring resonators made from different metals confirm the recent numerical simulations [Phys. Rev. E **65**, 036622 (2002)] that metamaterials exhibit permittivity-dependent resonant properties. In the terahertz regime, the measured inductive–capacitive resonance is found to strengthen with a higher ratio of the real to the imaginary parts of metal permittivity, and this remains consistent at various metal thicknesses. Furthermore, we found that metamaterials made even from a generally poor metal become highly resonant owing to a drastic increase in the value of the permittivity at terahertz frequencies. © 2008 Optical Society of America

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Recently, the unique properties, rich physics, and fascinating potential of metamaterials have inspired significant research over a broad range of the electromagnetic spectrum. Metamaterials are typically made from arrays of subwavelength resonator structures embedded in an insulating or dielectric boarding medium. They behave as continuous materials and enable intriguing electromagnetic phenomena, such as negative refraction, perfect lensing, and cloaking [1–3]. Recently, metamaterials have attracted increased attention in the terahertz regime, where they may advance the development of next-generation terahertz elements, such as integrated filters, modulators, detectors, and sensors [4–7].

Most metamaterials utilize split-ring resonators (SRRs) to achieve a desired response. The electric permittivity ($\varepsilon_m=\varepsilon_{rm}+i\varepsilon_{im}$) of metal SRRs is an important factor that is closely associated with the establishment of left-handed resonance in metamaterials [8]. In typical SRRs, the fundamental resonance is inductive capacitive (LC) in nature, where current circulates in the metallic loops to create inductance and charge accumulates at ring gaps to provide capacitance. The complex permittivity, $\varepsilon_m$, of the metal plays a very important role in making the structure highly resonant [9]. Although the exact values of $\varepsilon_m$ were not known in the gigahertz regime, a numerical study investigating the resonance properties of SRR-based microwave metamaterials has verified this dependence on $\varepsilon_m$ [8]. By fixing the real part of permittivity to $\varepsilon_{rm}=1$ and increasing the magnitude of the imaginary permittivity, the resonance gap of SRRs was found to become narrower and the left-handed effect was further enhanced.

At terahertz frequencies the values of metal permittivity are much higher than those at optical frequencies but slightly lower than those in the gigahertz regime. We present experimental and simulation results of the effect of metal permittivity on transmission properties of terahertz metamaterials made from double SRRs. At the LC resonance, 0.5 THz, the transmission amplitude and linewidth are shown to exhibit dependence on the permittivity of the constituent metals. The resonant transmission minimum is enhanced in correspondence with an increase in the imaginary part of the permittivity, $\varepsilon_{im}$, or an increase in the ratio of the real to the imaginary permittivity, $-\varepsilon_{rm}/\varepsilon_{im}$, showing consistency with recent numerical predictions at microwave frequencies [8]. The optimal transmission minimum and $Q$ factor are observed in SRRs made from Ag, a highly conducting metal, though $Q$ is effectively saturated owing to the radiative nature of the SRRs. More interestingly, metamaterials made from Pb, a generally poor metal, also exhibit strong LC resonances owing to a large permittivity in the terahertz regime. It thus indicates that terahertz metamaterials operate well over a wide range of constituent metals. This is essential in integrated terahertz components applications, such as filters and modulators, when fine control of resonant properties is needed with a fixed layout design.

Broadband terahertz time-domain spectroscopy (THz-TDS) was employed to characterize the metamaterial transmission properties [6,10]. Planar SRR metamaterials of three different metals—Pb, Al, and Ag—were lithographically fabricated on a silicon substrate (0.64 mm thick, $p$-type resistivity 20 Ω cm). The inset of Fig. 1(a) shows the diagram of a double SRR unit. The SRRs made from each metal are prepared with three different thicknesses—$0.5\delta$, $1\delta$, and $2\delta$—where $\delta$ is the skin depth at 0.5 THz. The calculated values of the skin depth are 336, 116, and 84 nm for Pb, Al, and Ag, respectively. Figures 1(a)–1(c) show the measured amplitude transmission of the SRR metamaterials made from different metals [6]. When the planar SRRs are $0.5\delta$ thick, as shown in Fig. 1(a), a well-defined LC resonance develops at 0.5 THz. The difference in resonance strength for different metals is clearly seen, with Ag

 

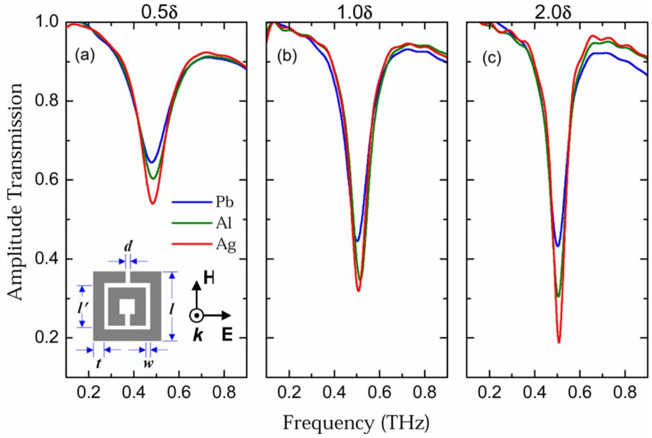


Fig. 1. (Color online) Measured frequency-dependent amplitude transmission of planar double SRR metamaterials made from Pb, Al, and Ag with various film thicknesses: (a) $0.5\delta$, (b) $1.0\delta$, and (c) $2.0\delta$, near the LC resonance. Inset, schematic of the double SRR unit with dimensions $w=3\ \mu$m, metal width $t=6\ \mu$m, $l=36\ \mu$m, effective length of the SRR unit $l'=21\ \mu$m, $d=2\ \mu$m, and the periodicity $P=50\ \mu$m.

SRRs featuring the deepest resonant transmission of 53.9%, while Al and Pb SRRs are limited at 60.2% and 64.4%, respectively.

The resonant behavior of the SRRs is further compared by using metals with different thicknesses. As shown in Figs. 1(b) and 1(c), by increasing the metal thickness to $1.0\delta$ and $2.0\delta$, the resonance is further strengthened for all metals. The Ag SRRs consistently reveal the strongest resonance, having a transmission minimum reduced to 18.5% at a thickness of $2.0\delta$. Comparatively, for the $2.0\delta$ thick Al and Pb samples the transmissions are 30.2% and 43.1%, respectively. The measured transmission minima at the LC resonance are plotted in Fig. 2(a) as a function of metal thickness. It is worth noting that when the metal thickness is varied from $1.0\delta$ to $2.0\delta$, the LC transmission for the Pb metamaterials is nearly saturated, while it is continuously strengthened for the Ag SRRs.

At 0.5 THz the complex permittivity of the given constituent metals are $\varepsilon_{\mathrm{Pb}}=-1.80\times10^3+1.65\times10^5 i$, $\varepsilon_{\mathrm{Al}}=-3.40\times10^4+1.34\times10^6 i$, and $\varepsilon_{\mathrm{Ag}}=-2.50\times10^5+2.15\times10^6 i$ [11,12]. The imaginary part of permittivity $\varepsilon_{im}$ shows a monotonic increase from Pb, Al, to Ag.

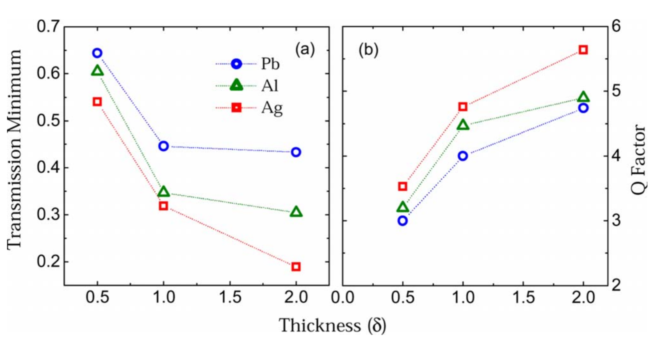


Fig. 2. (Color online) (a) Measured amplitude transmission minima and (b) $Q$ factor at the LC resonance 0.5 THz for different metal SRRs with various thicknesses in skin depth.

By noting the measured results shown in Fig. 1, the LC resonance for different constituent metals is seen to strengthen with increasing $\varepsilon_{im}$. Such a trend remains true at various given thicknesses, as shown in Figs. 1(a)–1(c). This is consistent with the recent numerical predictions using the transfer matrix method at microwave frequencies [8]. The electric permittivity of Ag, Al, and Pb can be well described by the Drude model [11]. At terahertz frequencies the Drude permittivity can be approximately given as $\varepsilon_m \approx -\sigma_{\mathrm{dc}}/(\varepsilon_0\Gamma)+i\sigma_{\mathrm{dc}}/(\varepsilon_0\omega)$, where $\sigma_{\mathrm{dc}}$ is the dc conductivity, $\Gamma=1/\tau$ is the damping rate with $\tau$ being the average collision time, and $\varepsilon_0$ is the vacuum permittivity [13]. Our THz-TDS results agree well with this simplified Drude expression in that $\varepsilon_{im}$ is proportional to the dc conductivity $\sigma_{\mathrm{dc}}$, suggesting that the LC resonance is more pronounced with metamaterials of higher conductivity [8,9].

Furthermore, when the contribution of $\varepsilon_{rm}$ is considered, the LC resonance is found to be enhanced with an increasing ratio, $-\varepsilon_{rm}/\varepsilon_{im}$, at terahertz frequencies. It was shown that a better conducting metal is characterized with a higher ratio $-\varepsilon_{rm}/\varepsilon_{im}$ [12,14]. The experimentally determined values $-\varepsilon_{rm}/\varepsilon_{im}$ for the constituent metals at each given metal thickness are plotted in Fig. 3 from 0.1 to 3.0 THz. At 0.5 THz, the ratios are 0.011, 0.025, and 0.116 for Pb, Al, and Ag, respectively [11,12]. As can be seen in Fig. 1, the measured LC resonance is indeed strengthened with the increasing ratio $-\varepsilon_{rm}/\varepsilon_{im}$ at each given metal thickness. This again is consistent with the simplified Drude permittivity, where the ratio $-\varepsilon_{rm}/\varepsilon_{im}$ is inversely proportional to the damping rate [11,15].

In addition, the quality ($Q$) factor of the LC resonance shows dependence on the permittivity of the constituent metals [16]. As shown in Fig. 2(b), the experimentally extracted $Q$ at the LC resonance shows an increasing trend with higher $\varepsilon_{im}$, as well as $-\varepsilon_{rm}/\varepsilon_{im}$. This relationship is limited, however, as the metal conductivity becomes very high. It is known that the SRR can be treated as an equivalent

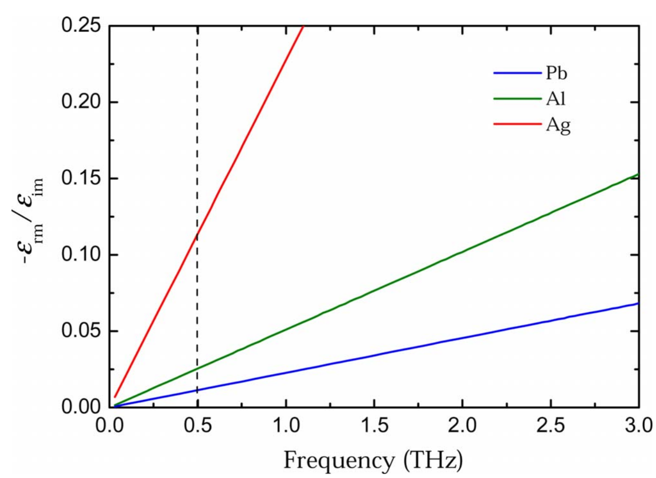


Fig. 3. (Color online) Frequency-dependent Drude ratio, $-\varepsilon_{rm}/\varepsilon_{im}$ of Pb, Al, and Ag in the terahertz regime [11,12]; the vertical dashed line indicates the LC resonance frequency 0.5 THz.

series resistor–inductor–capacitor (RLC) circuit. The $Q$ of the RLC series circuit is inversely proportional to the resistance, $Q \propto 1/R$, where $R$ is an effective resistance of the double SRR unit. However, SRRs are also radiators, continuously shedding resonant energy following excitation. Therefore the effective resistance can be expressed in two terms: the ohmic resistance, $R_L$, and the radiation resistance, $R_R$, representing energy loss by heating and by radiation, respectively. At terahertz frequencies, the ohmic resistance can be approximately given through the equivalent model, $R = 4l'/(th\omega\varepsilon_0\varepsilon_{im})$, with $h < 2\delta$, $h$ being the metal thickness [17]. The ohmic resistance decreases with increasing $\varepsilon_{im}$ or conductivity $\sigma_{\text{dc}}$ of metals, thus increasing the $Q$ factor. When the metal thickness approaches $2.0\delta$, the $Q$ of the Ag metamaterial shows a 19% increase as compared with the Pb SRRs. The $Q$ does not change as dramatically as $\varepsilon_{im}$ owing to the interplay of $R_L$ and $R_R$. As the conductivity increases, $R_L$ becomes negligible compared to $R_R$. Further increases in $\varepsilon_{im}$ or $\sigma_{\text{dc}}$ do not affect $R_R$ and should result in very little improvement in $Q$.

The experimental results were supplemented by finite-element simulations using CST Microwave Studio [7]. Figures 4(a) and 4(b) illustrate, respectively, the measured and simulated amplitude transmissions of 300 nm thick SRR metamaterials made from different metals. Both the measured and simulated LC resonances reveal similar permittivity dependent behavior. The measured transmission minima for the Pb, Al, and Ag samples are 47.7%, 25.5%, and 14.1%, respectively, showing a good agreement with the simulation results. The deviations in the simulated resonance frequencies may be caused by the approximations of defect-free SRRs with frequency-independent metal properties and thickness-independent conductivity, which are not necessarily true [18].

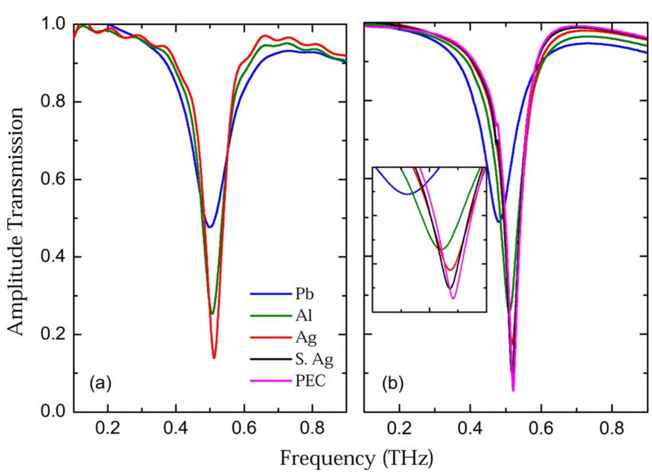


Fig. 4. (Color online) (a) THz-TDS results and (b) finite-element simulations of the LC resonant transmission of metamaterials made from 300 nm thick Pb, Al, Ag, S. Ag, and PEC. Inset, blow off of the simulated LC resonance.

It is important to note that the simulations enable an extrapolation of measured results. Figure 4(b) shows the simulated transmission for SRRs made from a hypothetical metal having conductivity ten times greater than that of the regular Ag, here referred to as super Ag (S. Ag), and SRRs made from a perfect electric conductor. The resonance strength increases as $\varepsilon_{im}$ increases and $R_L \to 0$, but the improvement in $Q$ is clearly saturated, verifying that radiation resistance limits $Q$ for SRRs of very low loss.

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